Olena Atlasiuk
Institute of Mathematics of the NAS & University of Helsinki, Ukraine & Finland

ODE–systems with boundary conditions containing higher order derivatives

This is a joint work with Prof. Volodymyr Mikhailets (Institute of Mathematics of the National Academy of Sciences of Ukraine, Kyiv, Ukraine).

We develop a general theory of solvability of linear inhomogeneous boundary-value problems for systems of ordinary differential equations of arbitrary order in Sobolev spaces. Boundary conditions are allowed to be overdetermined or underdetermined. They may contain derivatives, of the unknown vector-valued function, whose integer or fractional orders exceed the order of the differential equation. Similar problems arise naturally in various applications. The theory introduces the notion of a rectangular number characteristic matrix of the problem. The index and Fredholm numbers of this matrix respectively coincide with the index and Fredholm numbers of the inhomogeneous boundary-value problem. Unlike the index, the Fredholm numbers (i.e. the dimensions of the problem kernel and co-kernel) are unstable even for small (in the norm) finite-dimensional perturbations. We give examples in which the characteristic matrix can be explicitly found. We also prove a limit theorem for a sequence of characteristic matrices. Specifically, it follows from this theorem that the Fredholm numbers of the problems under investigation are semicontinuous in the strong operator topology. Such a property ceases to be valid in the general case.
The research is financially supported by the Academy of Finland, grant no. 359642.

References.

[1] V. MIKHAILETS, O. ATLASIUK, *The solvability of inhomogeneous boundary-value problems in Sobolev spaces*, Banach J. Math. Anal. 18(2), 12 (2024).